

In a nutshell: The shooting method

Given a boundary-value problem (BVP)

$$\begin{aligned}u^{(2)}(x) &= f(x, u(x), u^{(1)}(x)) \\ u(a) &= u_a \\ u(b) &= u_b\end{aligned}$$

This technique uses iteration, and at each step we find the solution to the initial-value problem (IVP) using an algorithm such as the Dormand-Prince method.

1. Let $s_0 \leftarrow \frac{u_b - u_a}{b - a}$ and approximation the solution $u_0(x)$ that is the solution to the IVP:

$$\begin{aligned}u^{(2)}(x) &= f(x, u(x), u^{(1)}(x)) \\ u(a) &= u_a \\ u^{(1)}(a) &= s_0\end{aligned}$$

If $|u_0(b) - u_b| < \varepsilon_{\text{abs}}$, we are done, and this approximation is the solution.

2. Otherwise, let $s_1 \leftarrow \frac{2u_b - u_0(b) - u_a}{b - a}$ and approximation the solution $u_1(x)$ that is the solution to the IVP:

$$\begin{aligned}u^{(2)}(x) &= f(x, u(x), u^{(1)}(x)) \\ u(a) &= u_a \\ u^{(1)}(a) &= s_1\end{aligned}$$

If $|u_1(b) - u_b| < \varepsilon_{\text{abs}}$, we are done, and this approximation is the solution.

3. Let $k \leftarrow 1$.
4. If $k > N$, we have iterated N times, so stop and return signalling a failure to converge.
5. Let $s_{k+1} \leftarrow \frac{s_{k-1}(u_k(b) - u_b) - s_k(u_{k-1}(b) - u_b)}{u_k(b) - u_{k-1}(b)}$, the solution to the secant method applied to last two values

$(s_{k-1}, u_{k-1}(b) - u_b)$ and $(s_k, u_k(b) - u_b)$, and approximation the solution $u_{k+1}(x)$ that is the solution to the IVP:

$$\begin{aligned}u^{(2)}(x) &= f(x, u(x), u^{(1)}(x)) \\ u(a) &= u_a \\ u^{(1)}(a) &= s_{k+1}\end{aligned}$$

If $|u_{k+1}(b) - u_b| < \varepsilon_{\text{abs}}$, we are done, and this approximation is the solution.

6. Increment k and return to Step 4.